Mathematical Logic

Lecture 6: From Syllogisms to Predicate Logic

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Reasoning about Predicates

Example

All humans are animals

All animals are mortal

All humans are mortal

Intuitively, this is a valid inference, but we cannot explain it using Propositional Logic.

Reasoning about Predicates

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$$\frac{\begin{array}{c}\text{All humans are animals}\\ \text{All animals are mortal}\end{array}}{\text{All humans are mortal}}$$

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Reasoning about Predicates

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Try to find the “variables“ and “logical constants“ in this argument.

Reasoning about Predicates

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Reasoning about Predicates

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$$\frac{\begin{array}{l}\text{All } A \text{ are } B \\ \text{All } B \text{ are } C\end{array}}{\text{All } A \text{ are } C}$$

Try to find the “variables“ and “logical constants“ in this argument.

Reasoning about Predicates

- The above example is known as a Syllogism .
  Aristotle (384 BC - 322 BC): *Prior Analytics*
- Formally, a syllogism always consists of two premises and one conclusion. None of them are compound propositions.

Reasoning about Predicates

- The components A, B, and C in the previous example are called predicates .
- A predicate is used to refer to a property . Properties have an intension and an extension .
- The extension is the class or set formed by the objects that have the property.

Reasoning about Predicates

Example

No society is static

All schools are societies

No school is static

Reasoning about Predicates

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Reasoning about Predicates

Example

All cars are movable

Some houses are cars

Some houses are movable

Reasoning about Predicates

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Reasoning about Predicates

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Reasoning about Predicates

Example

$$\frac{\begin{array}{l}\text{No homework is interesting}\\ \text{Some logic exercises are homework}\end{array}}{\text{Some logic exercises are not interesting}}$$

Reasoning about Predicates

Example

No homework is interesting

Some logic exercises are homework

Some logic exercises are not interesting

Reasoning about Predicates

- The parts of the premises and conclusions of a syllogism that are not variables (predicates) determine the type of those propositions. There are four types:

Affirmo	All A are B	Universal Affirmative
NEgo	No A are B	Universal Negative
AffIrmo	Some A are B	Particular Affirmative
NegO	Some A are not B	Particular Negative

- The terms "All" and "Some" are called the universal quantifier and the existential quantifier.

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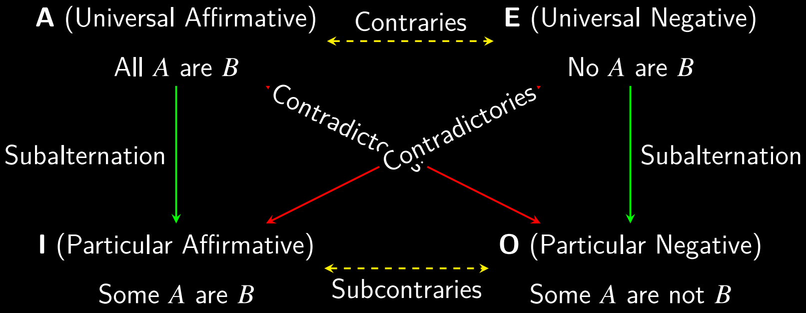
Reasoning about Predicates

Square of Opposition



Existential import

Reasoning about Predicates

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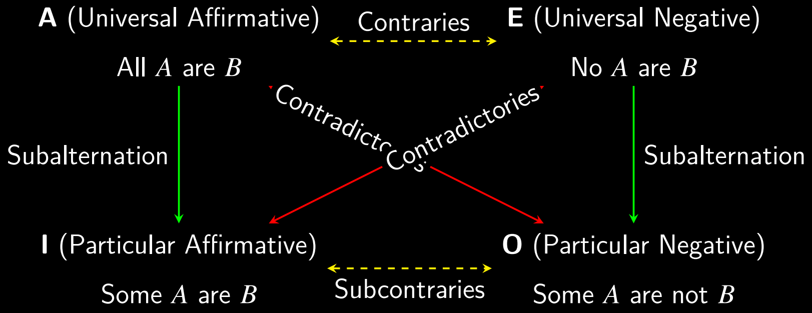

Existential import

Reasoning about Predicates

Example (Syllogism valid with existential import but invalid in modern logic)

All dragons are fire-breathers

All dragons are reptiles

Some reptiles are fire-breathers

Reasoning about Predicates

Example ("Existence" as a predicate)

God possesses all good properties

"Existence" is a good property

God exists

Problem: This is not a valid syllogism. "Existence" cannot be both a name and a predicate in a syllogism.

Reasoning about Predicates

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"Existence" is possessed by God

Problem: While structurally a valid syllogism now, it wrongly assumes "Existence" is a property (predicate).

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Reasoning about Predicates

Existence is not a property of things.

- "This is a red leaf" vs. "This is an existing red leaf"

  Kant: *Being ... is not a concept of something which could be added to the concept of a thing*

- "Sun Wukong does not exist"
- The modern logic approach: Existence is a property of properties (a second-order property); it is a quantifier.

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Reasoning about Predicates

Existence is not a property of (first-order) things.

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Reasoning about Predicates

Syllogistic Figures

- Usually, we use S to denote the Subject (the first predicate in the conclusion), P to denote the Predicate (the second predicate in the conclusion), and M for the predicate that only appears in the premises —the middle term.
- All conclusions are of the form:

$$\text{(All/Some) } S \text{ (are/are not) } P$$

Reasoning about Predicates

Syllogistic Figures

- A syllogism (its premises) can take one of 4 figures based on the position of the middle term:

	Figure 1	Figure 2	Figure 3	Figure 4
Major Premise	$M - P$	$P - M$	$M - P$	$P - M$
Minor Premise	$S - M$	$S - M$	$M - S$	$M - S$

Reasoning about Predicates

- We can identify the form of a syllogism by its Type + Figure: For example, AAA-1 is

$$\frac{\begin{array}{c}\text{All } M \text{ are } P\\ \text{All } S \text{ are } M\end{array}}{\text{All } S \text{ are } P}$$

Reasoning about Predicates

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$$\frac{\begin{array}{c}\text{All } M \text{ are } P\\ \text{No } S \text{ are } M\end{array}}{\text{No } S \text{ are } P}$$

Reasoning about Predicates

There are $4^3 = 64$ possible types, and with 4 figures, there are 256 possible syllogism forms. Not all forms are valid. We can judge if a syllogism is valid based on the following rules:

Reasoning about Predicates

- A term that is distributed (refer to the entire category) in the conclusion must also be distributed in the premises.
- The middle term must be distributed at least once.
- The number of negative propositions in the conclusion must equal the number of negative propositions in the premises.
- The number of particular propositions in the conclusion must equal the number of particular propositions in the premises.

Reasoning about Predicates

We need a more intuitive way to judge whether a syllogism is valid.

Operations on Sets or Classes

Notation

- We use $x \in A$ to indicate that the (object) x is an element of the (set) A, read as "x belongs to A".
- We use $\{x \mid \varphi(x)\}$ to denote the set of all objects having the property φ, which is the extension of the property φ.
- We can also denote a set by enumerating its elements: $\{a_1, a_2, \ldots\}$.

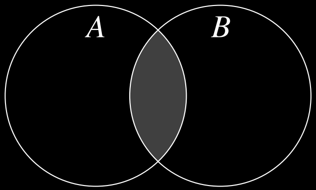
Operations on Sets or Classes

Binary Operations on Sets

- Intersection : $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$
- Union : $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$
- Difference : $A \setminus B = \{x \mid x \in A \text{ and } x \notin B\}$

Operations on Sets or Classes

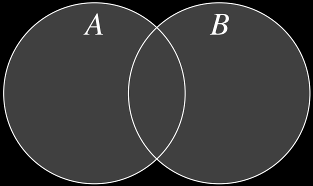
Venn Diagram: Intersection



Shaded area: $A \cap B$

Operations on Sets or Classes

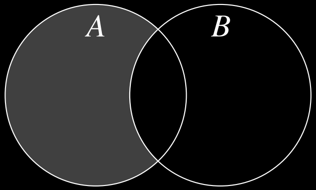
Venn Diagram: Union



Shaded area: $A \cup B$

Operations on Sets or Classes

Venn Diagram: Difference



Shaded area: $A \setminus B$

Operations on Sets or Classes

Definition (Subset Relation)

Given two sets A and B, we define $A \subset B$ (read as "A is a subset of B" or "A is included in B") if and only if all elements in A are also elements in B.

It is easy to see that:

- $A \cap B \subset A$
- $A \setminus B \subset A$
- $A, B \subset A \cup B$

Operations on Sets or Classes

Russell's Paradox

- A set itself can also be an element of a set. For example, the power set of X $P(X) = \{Y \mid Y \subset X\}$.
- If every property corresponds to a set, then $R = \{X \mid X \notin X\}$ would also be a set.
- Question: Does $R \in R$?

Operations on Sets or Classes

Usually, we talk about sets within a given domain of discourse (or simply domain) U:

$$\{x \in U \mid \varphi(x)\}$$

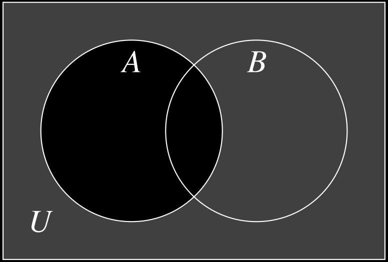
- Obviously, $\{x \in U \mid \varphi(x)\} \subset U$.
- Generally, it is assumed that no set belongs to itself, thus

$$\{x \in U \mid x \notin x\} = U \notin U$$

Operations on Sets or Classes

Given a domain U, we can define the complement of a set:

$$\overline{A} = A^{C} = \{x \in U \mid x \notin A\}$$



Operations on Sets or Classes

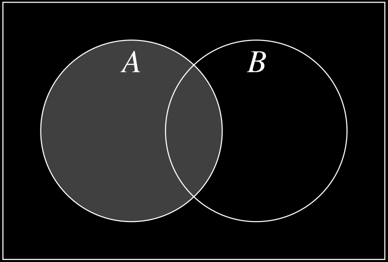
Some set operations can be defined in terms of each other.

For example, we can use intersection and complement to define other operations:

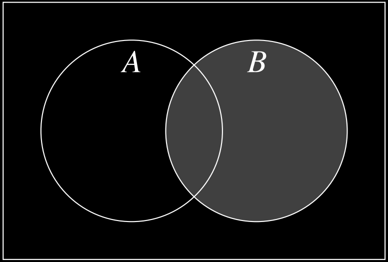
- $A \setminus B = A \cap \overline{B}$
- $A \cup B = \overline{\overline{A} \cap \overline{B}}$

Operations on Sets or Classes

Calculating $\overline{\overline{A} \cap \overline{B}}$ using Venn diagrams



A



B

Operations on Sets or Classes

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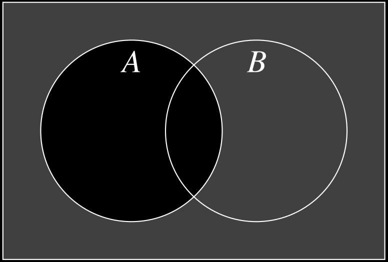
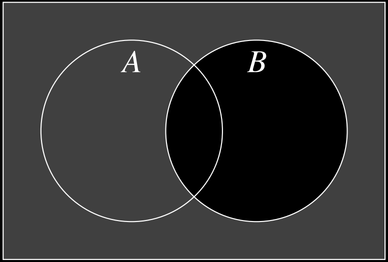


$\overline{A}$



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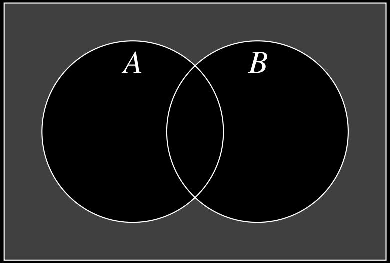

$\overline{A} \cap \overline{B}$

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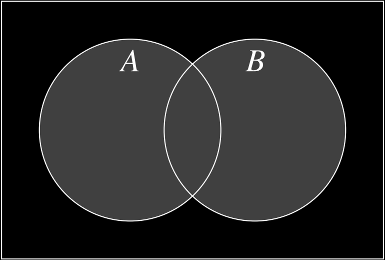

$$\overline{\overline{A} \cap \overline{B}} = A \cup B$$

Operations on Sets or Classes

Similarly, we can derive:

- $\overline{\overline{A}} = A$ (Double Complement)
- $\overline{A \cup B} = \overline{A} \cap \overline{B}$, $\overline{A \cap B} = \overline{A} \cup \overline{B}$ (De Morgan's Laws)
- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (Distributivity)
- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (Distributivity)

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Operations on Sets or Classes

Set Operations and Propositional Logic

- $A \cap B = \{x \in U \mid x \in A \land x \in B\}$
- $A \cup B = \{x \in U \mid x \in A \lor x \in B\}$
- $\overline{A} = \{x \in U \mid \neg(x \in A)\}$
- $\{x \in U \mid x \in A \to x \in B\} = ?$
- $A \setminus B = \{x \in U \mid \neg(x \in A \to x \in B)\} = \overline{\overline{A} \cup B} = A \cap \overline{B}$

Operations on Sets or Classes

Set Operations and Propositional Logic

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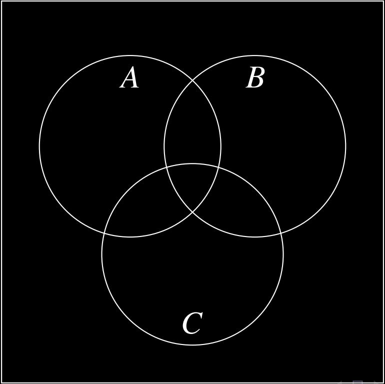
Judging Validity with Venn Diagrams

Recall: An inference is valid if and only if in every situation where the premises are true, the conclusion is also true.

What are the possible situations in a syllogism?

Judging Validity with Venn Diagrams

Every standard syllogism involves 3 predicates. We need to consider the following Venn diagram. The box is partitioned into ? minimal regions.



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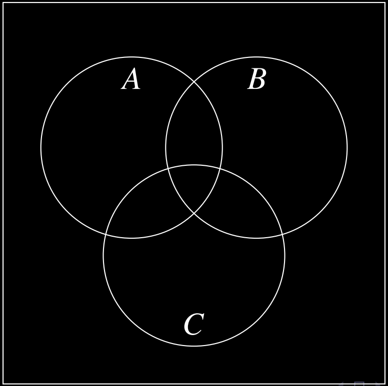

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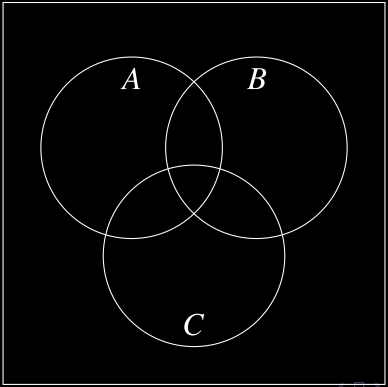

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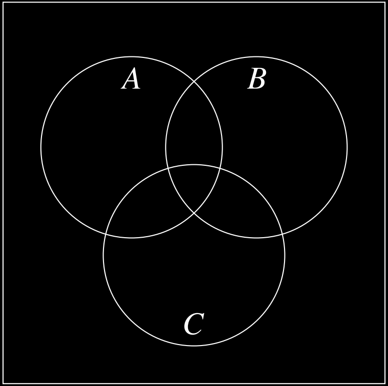
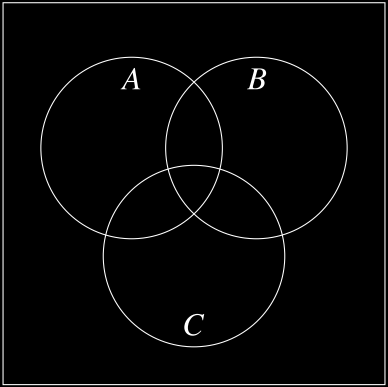


Judging Validity with Venn Diagrams

Can you represent these cells using set operations? What about regions formed by multiple cells?



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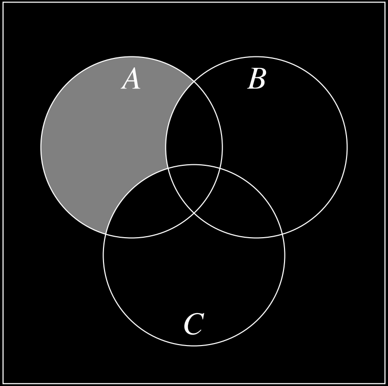

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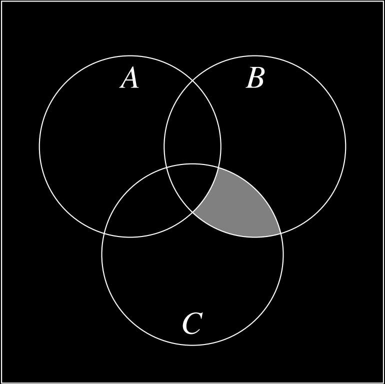

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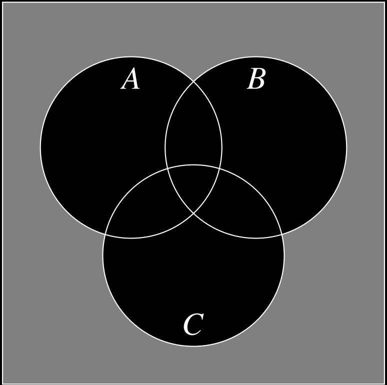

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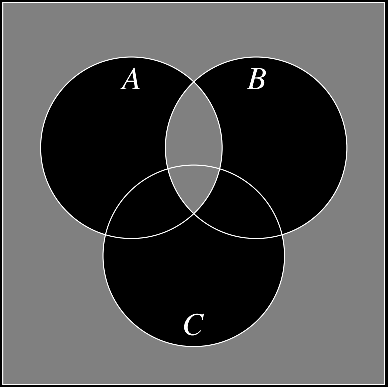
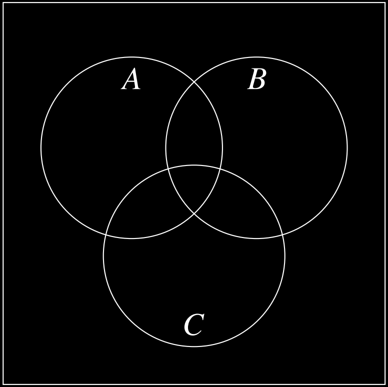


Judging Validity with Venn Diagrams

What does "All A are B" say about the Venn diagram below?

What about "No A are B"? "Some A are B"? "Some A are not B"?



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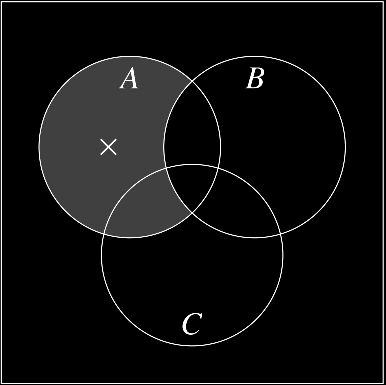

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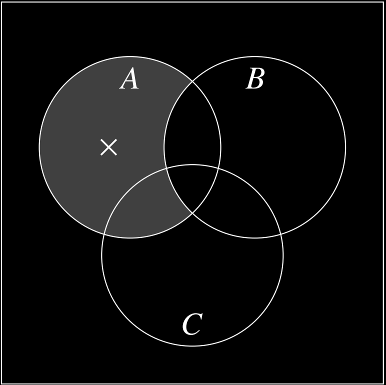

Judging Validity with Venn Diagrams

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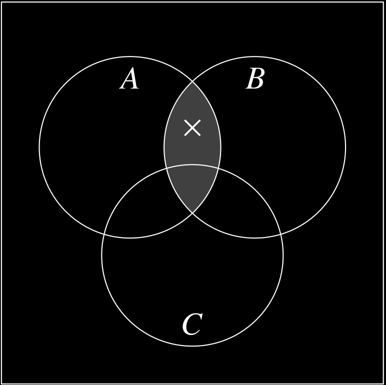

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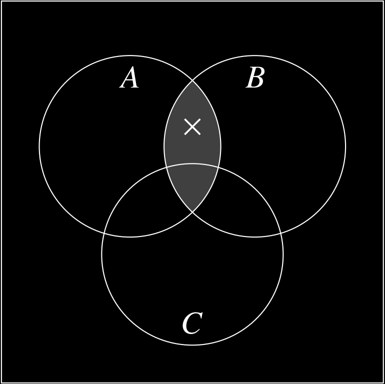

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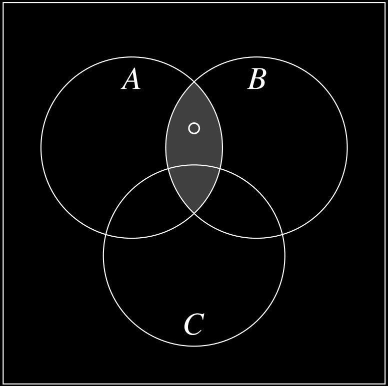

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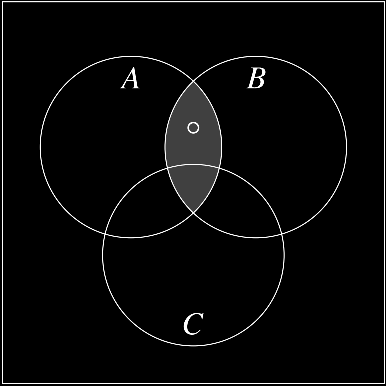

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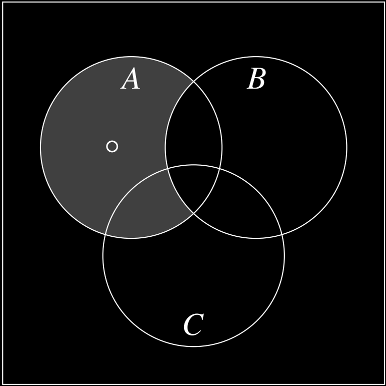


Judging Validity with Venn Diagrams

- Whether it is A, E, I, or O, they all talk about whether certain cells in the Venn diagram are empty or non-empty.
- Considering whether each of the 8 cells is empty or non-empty, we can distinguish $2^8 = 256$ basic situations .
- Valid syllogism: In every basic situation, if both premises are satisfied, then the conclusion is also satisfied.

Judging Validity with Venn Diagrams

Try to judge the validity of the following syllogism using Venn diagrams

All politicians are liars

No students are liars

No students are politicians

Judging Validity with Venn Diagrams

Try to judge the validity of the following syllogism using Venn diagrams

All politicians are liars

No students are politicians

No students are liars

Judging Validity with Venn Diagrams

Try to judge the validity of the following syllogism using Venn diagrams

Some philosophers are Greek citizens

No Greek citizens are slaves

No philosophers are slaves

Judging Validity with Venn Diagrams

Try to judge the validity of the following syllogism using Venn diagrams

No Greek citizens are slaves

No slaves are philosophers

No Greek citizens are philosophers

Judging Validity with Venn Diagrams

Try to judge the validity of the following syllogism using Venn diagrams

No Greek citizens are slaves

Some slaves are philosophers

Some philosophers are not Greek citizens

More Predicates

Example

All A are B

No C are B

Some C are D

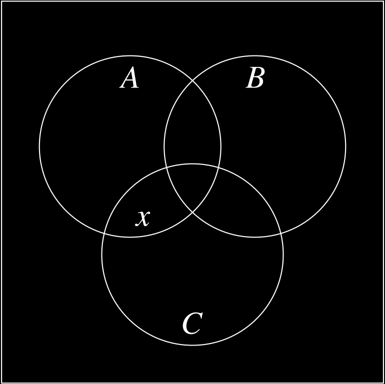
Some D are not A

Syllogisms and Propositional Logic

Recall: In a Venn diagram, if we consider 3 predicates, we can partition the box into 8 cells. Considering a single object x, we can abbreviate x satisfies A (i.e., $A(x)$) with the propositional symbol a, $B(x)$ with b, and $C(x)$ with c. In this case, which cell x belongs to gives a valuation of the three propositional symbols a, b, c.

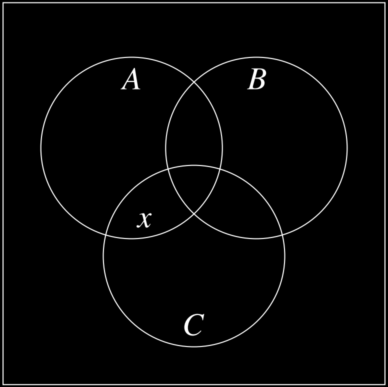
Syllogisms and Propositional Logic

For example, the position of x in the following Venn diagram corresponds to $V(a) =?, V(b) =?, V(c) =?$, or simply $a\bar{b}c$.



Syllogisms and Propositional Logic

For example, the position of x in the following Venn diagram corresponds to $V(a) = 1, V(b) = 0, V(c) = 1$, or simply $a\bar{b}c$.



Syllogisms and Propositional Logic

- Similarly, the Venn diagram for 4 predicates has 16 cells.
- The Venn diagram for n predicates has 2^n cells.
- As established, which cell an object occupies in the Venn diagram corresponds to an assignment of truth values to a set of propositional variables.

Syllogisms and Propositional Logic

Back to the problem of judging the validity of a syllogism:

- A syllogism with premises φ and ψ and conclusion χ is valid, denoted as $\varphi, \psi \vDash \chi$, if and only if $\{\varphi, \psi, \neg\chi\}$ is unsatisfiable.
- Judging whether a syllogism $\varphi, \psi \vDash \chi$ is valid is equivalent to judging whether the set of propositions $\{\varphi, \psi, \neg\chi\}$ is satisfiable.
- If χ is a universal proposition, then $\neg\chi$ is a particular proposition; and vice versa.

Syllogisms and Propositional Logic

- Any three universal propositions (whether affirmative or negative) are always satisfiable. Why?
- Any three particular propositions are also satisfiable.
- $\{\varphi, \psi, \neg\chi\}$ is unsatisfiable only when a particular proposition asserts that some cell(s) must contain an object, which contradicts other propositions asserting that those same cell(s) are empty.

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Syllogisms and Propositional Logic

In a syllogism,

- A universal proposition always rules out 2 cells .
- A particular proposition always asserts that at least one object exists in a combination of two cells.
- In the syllogism $\varphi, \psi \models \chi$, each proposition concerns 2 cells that are different from those of the other. Therefore, a set $\{\varphi, \psi, \neg\chi\}$ containing two particular propositions and one universal proposition is always satisfiable.

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Syllogisms and Propositional Logic

- Therefore, only the case with one particular and two universal propositions among $\{\varphi, \psi, \neg\chi\}$ can be unsatisfiable.

Recall: "The number of particular propositions in the conclusion equals the number of particular propositions in the premises."

Syllogisms and Propositional Logic

- To judge whether a syllogism is valid, we only need to consider the case where $\{\varphi, \psi, \neg\chi\}$ contains exactly one particular proposition and two universal propositions.
- In these cases, if $\{\varphi, \psi, \neg\chi\}$ is satisfiable, there exists a situation with exactly one object that satisfies it. Why?
- Therefore, to judge if a syllogism is valid, we only need to consider the corresponding set $\{\varphi, \psi, \neg\chi\}$ having one particular and two universal propositions, and a domain U with at most one object.

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Syllogisms and Propositional Logic

Suppose $\{\varphi, \psi, \neg\chi\}$ contains exactly one particular and two universal propositions, and the only possible object in the domain is x. Recall that we use a for $A(x)$, b for $B(x)$, and c for $C(x)$.

- "All A are B" becomes $a \to b$
- "Some A are B" becomes $a \wedge b$
- "No A are B" becomes $a \to \neg b$
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Syllogisms and Propositional Logic

Thus, the problem of whether a syllogism is valid is reduced to whether a set of propositional logic formulas is satisfiable.

Example

Some A are B	is valid if and only if
No B are C	$\{a \wedge b,\ b \rightarrow \neg c,\ \neg(a \wedge \neg c)\}$
Some A are not C	is not satisfiable.

Exercises & Discussion

What is the middle term of the following syllogism?

No homework is interesting

Some logic exercises are homework

Some logic exercises are not interesting

Exercises & Discussion

Try to judge the validity of the following syllogism using Venn diagrams

Some philosophers are Greek citizens

No Greek citizens are slaves

No philosophers are slaves

Exercises & Discussion

Try to judge the validity of the following syllogism using Venn diagrams

No Greek citizens are slaves

No slaves are philosophers

No Greek citizens are philosophers

Exercises & Discussion*

- Try to find as many forms of categorical syllogisms as possible that are valid in Aristotle's system (assuming existential import) but invalid in modern logic.

Exercises & Discussion

Try to judge the validity of the following syllogism using Venn diagrams

No Greek citizens are slaves

Some slaves are philosophers

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Exercises & Discussion*

- Can you describe a computational procedure/program to judge the validity of a syllogism?
- Suppose the universal quantifier is interpreted with existential import. That is, "All A are B" implies "Some A are B". How would you modify the method for judging the validity of a syllogism?

Exercises & Discussion*

Using the subset relation $A \subset B$ as our primitive concept, try to define the following sets or operations without using the membership relation $\in$:

- The empty set $\emptyset$
- $A \cap B$, $A \cup B$, $A \setminus B$

Exercises & Discussion*

Consider "tetrasyllogisms", which involve four predicates and are always composed of three premises $\varphi_1, \varphi_2, \varphi_3$ and one conclusion ψ. Give further restrictions on the structure of tetrasyllogisms, and prove that:

- In a valid tetrasyllogism, there is exactly one particular proposition in $\{\varphi_1, \varphi_2, \varphi_3, \neg\psi\}$.
- Show that if $\{\varphi_1, \varphi_2, \varphi_3, \neg\psi\}$ contains exactly one particular proposition and is satisfiable, then there exists a situation with exactly one object that satisfies it.