Mathematical Logic

Lecture 1: Introduction to the Course and Logic

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Fall 2026

Course Information

- Time and Location:
  - Monday, Tuesday, Thursday, 1:00 PM –3:35 PM
  - H3201
- Webpage:

  `https://web.yangruizhi.cyou/logic2026/`
- Textbook or lecture notes: Slides can be download from the webpage

Our team

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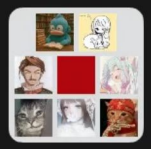
Evaluation

- Exercises
  - Formalization of natural language reasoning
  - Problem sets and in-class discussions
- Final Exam: Closed-book (based on the problem sets after each lecture)
  - Time: August 6
  - Location: In-class

Join the Wechat Group



群聊：Mathematical Logic-FISS 2026

该二维码7天内(7月18日前)有效，重新进入将更新

About This Course

Current Plan

- **Lectures:** Introduction to Logic
  - What, Why and How
- **Discussions:**
  - Applications
  - Problem sets

What is Logic?

- What is logic?
- What is the study of logic?
- What is the purpose of this course?

Logic in Everyday Life

Example (Waiters serving food)

Suppose you (C) and two friends (A and B) order food at a restaurant counter. You order fish-flavored shredded pork with rice. Friend A orders Yangzhou fried rice. Friend B orders pork liver noodles. You all sit down at a table. Soon, a waiter arrives with the three dishes.

Logic in Everyday Life

Example (Waiters serving food)

The waiter asks, "Who ordered the pork with rice?" You raise your hand, and the waiter places it in front of you. The waiter then asks, "Who ordered the fried rice?" Friend A signals. The waiter then gives the fried rice to A, and without asking, gives the noodles to B.

Logic in Everyday Life

Example (Phone not charging)

Your phone charger is plugged into the wall, the USB cable is connected to your phone, but the phone is not charging.

Logic in Everyday Life

Example (Phone not charging)

- You borrow your roommate's charger block, but your phone still does not charge.
- You use your roommate's phone with your cable and charger block, and their phone does not charge either.
- Conclusion: You buy a new USB cable.

Logic in Everyday Life

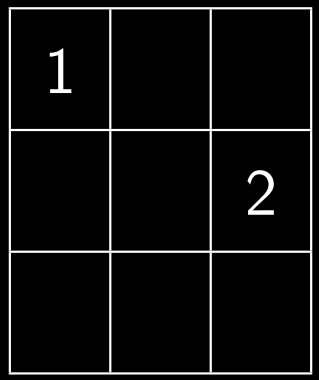
Example (3×3 Sudoku)

Rules:

- Fill each square with a number from 1 to 3.
- No row or column can contain duplicate numbers.

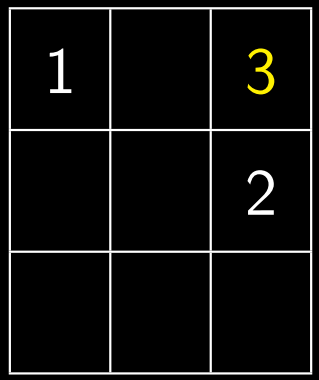
Logic in Everyday Life

Example (3×3 Sudoku)



Logic in Everyday Life

Example (3×3 Sudoku)



Logic in Everyday Life

Example (3×3 Sudoku)

1		3
3		2

Logic in Everyday Life

Example (3×3 Sudoku)

1	2	3
3		2
		1

Logic in Everyday Life

Example (3×3 Sudoku)

1	2	3
3	1	2
2		1

Logic in Everyday Life

Example (3×3 Sudoku)

1	2	3
3	1	2
2	3	1

Extracting the Logical Form

What do these three examples have in common?

- Pork Rice or Fried Rice or Noodles. not Pork Rice, not Fried Rice $\Rightarrow$ Noodles.
- Power broken or Cable broken or Phone broken. not Power broken, not Phone broken $\Rightarrow$ Cable broken.
- 1 or 2 or 3. not 1, not 2 $\Rightarrow$ 3.

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AI is not a logic machine

Example (LLM on Disjunctive Syllogism)

Prompt: The keys are either in the kitchen, the bedroom, or the living room. They are not in the kitchen. They are not in the bedroom. They are not in the living room either. Where are the keys?

LLM (in early days): Since the keys are not in those three rooms, they must be in the bathroom or in your car.

The Ambiguity of Natural Language

Example (Every boy loves a certain girl)

What does this sentence mean?

1. There is one specific girl (e.g., a famous pop star) whom *every* boy loves.
2. For each boy, there is some girl he loves (e.g., his own mother).

Why Formalize?

- Just like physics studies the laws of the physical world, logic studies the laws of reasoning.
- By formalizing logic, we strip away distracting details (like noodles or USB cables) to reveal the pure structure.
- Formalization prevents ambiguity and allows us to calculate truth just like we calculate numbers.

A Brief History of Logic

Like many disciplines, a long history can be written about logic, which can start from Egypt, Persia, India. But the key steps that really have an impact on today's logic are actually not many.

A Brief History of Logic

Ancient Greek Logic

- **Megarian-Stoic School**: Debated the truth conditions of conditional sentences (Early Propositional Logic).

  Philo (Diodorus's student, Zeno's friend) said that a conditional is true when it does not begin with a true proposition and finish with a false one ... Diodorus, on the other hand, says that a conditional is true if it neither was nor is possible that its antecedent is true and its consequent false

A Brief History of Logic

Ancient Greek Logic

- Aristotle (384-322 BC): Syllogism - term logic (primitive predicate logic)

  Mainly reflected in Aristotle's book *Organon*, which means instrument or tools

A Brief History of Logic

Modern Logic

- **Leibniz**: Vision of a universal language (*Characteristica universalis*).

  "The only way to rectify our reasonings is to make them as tangible as those of the Mathematicians, so that... when there are disputes among persons, we can simply say: Let us calculate, without further ado, to see who is right."

  The Art of Discovery (1685)

A Brief History of Logic

- **George Boole (1815-1864)**: Algebraic tradition — Boolean Logic.
- **Gottlob Frege (1848-1925)**: Invented modern Predicate Logic, combining propositional logic and Aristotelian logic.
- **David Hilbert**: Syntax and Proof Theory.
- **Alfred Tarski**: Semantics and Model Theory.
- **Kurt Gödel**: Completeness and Incompleteness Theorems.

A Brief History of Logic

"His [Leibniz's] idea of a universal characteristic... was too gigantic for the attempt to realize it to be able to go beyond the bare preliminaries. ... Even if this worthy goal cannot be reached in one leap, we need not despair of a slow, step-by-step approach. When a problem appears to be unsolvable in its full generality, one should temporarily restrict it... The ideography proposed here adds a new one to these fields, indeed the central one, which borders on all the others."

Gottlob Frege, *Begriffsschrift* (1879)

A Brief History of Logic

"In fact, we have truly mastered the theory of a natural science only when we have extracted its mathematical kernel and completely laid it bare."

David Hilbert (Königsberg address, 1930)

Philosophy of Logic

What exactly is logic studying?

- Is logic about truth?
  - What is truth? (Correspondence theory vs. Provability)
- Is logic about language?
  - Is it descriptive (how we actually speak) or normative (how we *should* speak)?
- Or is logic simply a tool applied across different domains?

Philosophy of Logic

Different philosophical views lead to different logical systems!

Example

Is the Law of Excluded Middle ($p \vee \neg p$) always valid?

- If truth is about correspondence to reality $\Rightarrow$ Yes (Classical Logic).
- If truth requires a constructive mathematical proof $\Rightarrow$ No (Intuitionistic Logic).

What is a Logical Truth?

Which of the following are logical truths?

- $p \rightarrow p$
- $a = a$
- $0 \neq 1$
- $\mathsf{e} = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$
- "e is an irrational number"
- $E = mc^2$

A Classical Puzzle: Knights and Knaves

- You arrive at a fork in the road. One path leads to safety, the other to a trap.
- Two guards stand at the fork. One is a Knight (always tells the truth), the other is a Knave (always lies).
- You don't know who is who.
- **Task**: You can only ask one yes/no question to one guard. How do you find the safe path?

A Classical Puzzle: Formalization

Let AK mean "A is a Knight" and BK mean "B is a Knight".

For any proposition φ, let $A(\varphi)$ mean "A asserts φ".

- $AK \leftrightarrow \neg BK$ (One is a knight, one is a knave)
- $AK \rightarrow (A(\varphi) \rightarrow \varphi)$ (Knights tell the truth)
- $\neg AK \rightarrow (A(\varphi) \rightarrow \neg\varphi)$ (Knaves lie)

Let p be the proposition "The left path is safe."

We ask Guard A: "If I ask Guard B whether the left path is safe, what would he say?"